

Further Pure 1

Chapter 4 Inequalities with Graph Sketching Practice

Easier: Linear and Simple Rational Functions

- On the same set of axes, sketch the graphs of $y = x$ and $y = \frac{4}{x}$.
 - Find the coordinates of the points of intersection of $y = x$ and $y = \frac{4}{x}$.
 - Hence write down the solution to the inequality $x \geq \frac{4}{x}$.
- $f(x) = x + 2$ and $g(x) = \frac{3}{x}$, $x \neq 0$.
 - Sketch $y = f(x)$ and $y = g(x)$ on the same set of axes.
 - Solve $f(x) = g(x)$.
 - Hence, or otherwise, solve the inequality $f(x) < g(x)$.
- On the same set of axes, sketch the graphs of $y = \frac{2}{x}$ and $y = \frac{1}{x-2}$.
 - Find the points of intersection of the two graphs.
 - Write down the solution to the inequality $\frac{2}{x} > \frac{1}{x-2}$.
- On the same set of axes, sketch the graphs of $y = 2x - 1$ and $y = \frac{2}{x-2}$.
 - Find the exact coordinates of the points of intersection of $y = 2x - 1$ and $y = \frac{2}{x-2}$.
 - Solve the inequality $2x - 1 \leq \frac{2}{x-2}$.
- $f(x) = \frac{4}{x^2}$, $x \neq 0$ and $g(x) = x + 3$.
 - Sketch $y = f(x)$ and $y = g(x)$ on the same set of axes.
 - Find the coordinates of the points where the graphs intersect.
 - Hence write down the solution to the inequality $f(x) < g(x)$. Give your answer using set notation.

Moderate: Rational vs. Rational (Linear Denominators)

- On the same set of axes, sketch the graphs of $y = \frac{x+1}{x-2}$ and $y = 3$.
 - Find the point of intersection of $y = \frac{x+1}{x-2}$ and $y = 3$.
 - Solve the inequality $\frac{x+1}{x-2} \geq 3$.
- On the same set of axes, sketch the graphs of $y = \frac{2x}{x+1}$ and $y = x$.
 - Find the points of intersection of $y = \frac{2x}{x+1}$ and $y = x$.
 - Hence, or otherwise, solve the inequality $\frac{2x}{x+1} > x$.
- $f(x) = \frac{2}{x-1}$, $x \neq 1$ and $g(x) = \frac{x}{x+2}$, $x \neq -2$.
 - Sketch $y = f(x)$ and $y = g(x)$ on the same set of axes.
 - Solve $f(x) = g(x)$, hence write down the solution to the inequality $f(x) \leq g(x)$.

9. a) On the same set of axes, sketch the graphs of $y = \frac{x}{x+3}$ and $y = \frac{2}{x+1}$.
 b) Find the exact coordinates of the points of intersection.
 c) Solve the inequality $\frac{x}{x+3} < \frac{2}{x+1}$. Give your answer using set notation.
10. a) On the same set of axes, sketch the graphs of $y = \frac{2(x-2)}{x+1}$ and $y = \frac{x-2}{x-1}$.
 b) Find the points of intersection of $y = \frac{2(x-2)}{x+1}$ and $y = \frac{x-2}{x-1}$.
 c) Write down the solution to the inequality $\frac{2(x-2)}{x+1} \geq \frac{x-2}{x-1}$.

Harder: Squared Denominators, Complex Rational Expressions, and Challenge Problems

11. $f(x) = \frac{3}{x^2}$, $x \neq 0$ and $g(x) = \frac{4-x}{x}$, $x \neq 0$.
 a) Sketch $y = f(x)$ and $y = g(x)$ on the same set of axes.
 b) Solve $f(x) = g(x)$.
 c) Hence, or otherwise, solve the inequality $f(x) > g(x)$.
12. a) On the same set of axes, sketch the graphs of $y = \frac{4}{x^2-9}$ and $y = \frac{1}{x-3}$.
 b) Find the point of intersection of the two graphs.
 c) Solve the inequality $\frac{4}{x^2-9} \leq \frac{1}{x-3}$.
13. a) On the same set of axes, sketch the graphs of $y = \frac{x}{4-x}$ and $y = \frac{4x}{(x-1)^2}$.
 b) Find the exact points of intersection of $y = \frac{x}{4-x}$ and $y = \frac{4x}{(x-1)^2}$.
 c) Hence, or otherwise, solve the inequality $\frac{x}{4-x} \geq \frac{4x}{(x-1)^2}$.
14. a) On the same set of axes, sketch the graphs of $y = x - 1$ and $y = \frac{4(1-x)}{(x+1)(x-4)}$.
 b) Find the points of intersection of $y = x - 1$ and $y = \frac{4(1-x)}{(x+1)(x-4)}$.
 c) Write down the solution to the inequality $x - 1 \leq \frac{4(1-x)}{(x+1)(x-4)}$.
15. a) On the same set of axes, sketch the graphs of $y = \frac{x^2}{x-2}$ and $y = x + 3$. (*Note: This is beyond the specification. This graph has an oblique asymptote—these occur when the order of the numerator is greater than the order of the denominator. Divide x^2 by $x - 2$ to find the oblique asymptote.*)
 b) Find the point of intersection of the two graphs.
 c) Solve the inequality $\frac{x^2}{x-2} < x + 3$. Give your answer using set notation.