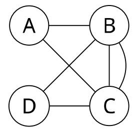
Solutions and Mark Scheme

1. (a) A finite sequence of edges such that the end vertex of one edge is the start vertex of the next.
   (b) A graph with no loops and no multiple edges between the same pair of vertices.
2. By the Handshaking Lemma, the sum of degrees must be even ($2\times$ edges). The sum of the known degrees is 28. Since 28 is even, x must be even to keep the total sum even.
3. Number of edges in $K_n = \frac{n(n-1)}{2}$. For K_{12}: $\frac{12\times11}{2} = 66$.
4. An adjacency matrix records the number of direct edges between vertices. A distance matrix (or weight matrix) records the weight (cost/distance) of the edges.
5. (a) Diagram should show 4 vertices with every vertex connected to every other vertex.
   (b) A cycle that visits every vertex in the graph exactly once (and returns to the start).
   (c) Example: $A-B-C-D-A$.
6. (a) It contains a multiple edge between vertices C and F.
   (b) A:3, B:2, C:3, D:4, E:2, F:4.
   (c) Distance Matrix:

$$\begin{pmatrix} & X & Y & Z \\ X & - & 5 & 12 \\ Y & 5 & - & 8 \\ Z & 12 & 8 & - \end{pmatrix}$$

7. **Graph 1 and Graph 2 are isomorphic.**
   - **Reasoning:** Both Graph 1 (Pentagon) and Graph 2 (Star) are **2-regular** (every vertex has a valency of 2). They are both single cycles of length 5.
   - Graph 3 (House with chord) has two vertices of valency 2 (bottom corners) and two vertices of valency 3 (top corners of square). Because the valencies do not match Graphs 1 and 2, it cannot be isomorphic to them.

8. 

(Labels A, B, C, D correspond to the matrix rows/cols. Note the double edge between B and C).

9. (a) Many answers possible. e.g., $P-Q-R-P$ or $P-S-R-P$.
   (b) A tree cannot contain any cycles. Since we identified a cycle, it is not a tree.
10. Drawing the Hamiltonian cycle as a regular polygon creates a clear "boundary" that separates the plane into an "inside" region and an "outside" region. This allows us to systematically check if remaining edges can be placed without crossing.
11. (a) 5 edges $(n-1)$.
    (b) A tree contains no cycles, so edges can never cross or enclose regions in a way that forces crossings.
12. (a) $A-B-C-D-E-F-A$.
    (b) AC, AD, BF, BE.

(c) **Planar**. Edges AC and BE cross, so one must move outside. Edges AD and BF do not cross AC or BE directly in a way that prevents resolving the conflict. Example valid configuration: AC Inside, BF Inside, AD Inside, BE Outside. Check: BE (Outside) connects 2-5. BF (Inside) connects 2-6. No conflict. AC (Inside 1-3) and BF (Inside 2-6) cross? Vertex 2 is in range 1-3. Vertex 6 is in range 3-1. Yes, they cross. So BF must be Outside. New Config: AC (In), AD (In), BE (Out), BF (Out). BE (Out 2-5) vs BF (Out 2-6). No cross (share B). AC (In 1-3) vs AD (In 1-4). No cross (share A). Therefore, graph is Planar.