Revision Worksheet: Graph Theory

Topic: Graphs, Networks, and Planarity

Questions

1. Define the following terms in the context of graph theory:
   (a) A **Walk**
   (b) A **Simple Graph**

[Total: 2 marks]

2. A simple graph G has 8 vertices. The degrees (valencies) of the vertices are $3, 3, 4, 4, 4, 5, 5, x$.
   (a) Explain why x must be an even number.

[Total: 1 mark]

3. Calculate the total number of edges in the complete graph K_{12}. State the formula used.

[Total: 2 marks]

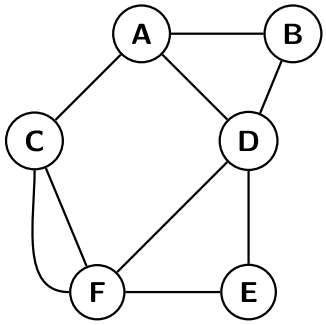
4. Explain the difference between an Adjacency Matrix and a Distance Matrix.

[Total: 2 marks]

5. (a) Draw the complete graph K_4.
   (b) Define a **Hamiltonian Cycle**.
   (c) Write down a Hamiltonian Cycle for K_4 using the vertices A, B, C, D.

[Total: 5 marks]

6. Consider **Graph A** below:



(a) Explain why Graph A is **not** a simple graph.

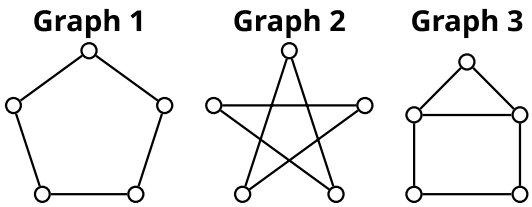
(b) List the valency of each vertex.

(c) The table below shows the distances (weights) between three towns X, Y, and Z. Draw the distance matrix for this network.

- X to Y: 5 miles
- Y to Z: 8 miles
- X to Z: 12 miles

[Total: 5 marks]

7. Look at the three graphs below:



Identify which two graphs are **isomorphic** and justify your answer by referring to the valencies (degrees) of the vertices.

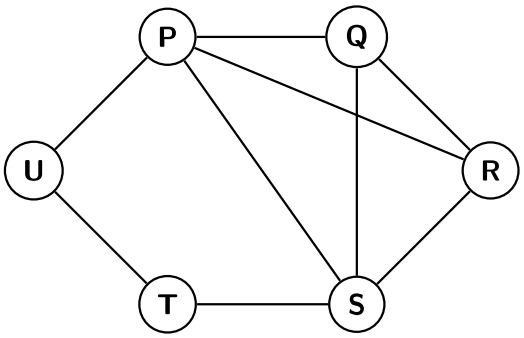
[Total: 3 marks]

8. A graph is represented by the following adjacency matrix M. Draw the graph represented by this matrix.

	A	B	C	D
A	0	1	1	0
B	1	0	2	1
C	1	2	0	1
D	0	1	1	0

[Total: 3 marks]

9. Consider **Graph B** below:



(a) Identify a cycle of length 3 in Graph B.

(b) Explain why this graph cannot be a tree.

[Total: 4 marks]

10. The first step of the **Planarity Algorithm** involves finding a Hamiltonian cycle. Explain why the Hamiltonian cycle is drawn as a regular polygon (a circle of edges) in this algorithm.

[Total: 2 marks]

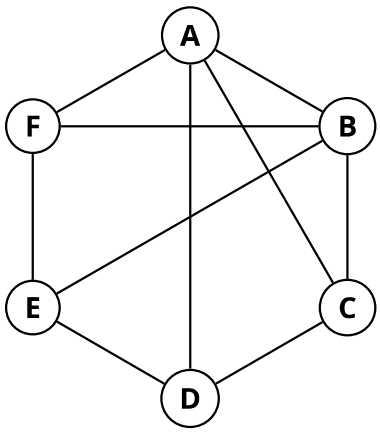
11. Draw a tree with 6 vertices.

(a) State the number of edges.

(b) Explain why a tree is always a planar graph.

[Total: 4 marks]

12. Apply the Planarity Algorithm to the graph shown below.



(a) Identify the Hamiltonian cycle used in the drawing above.

(b) List the edges that are currently drawn inside the cycle.

(c) Determine whether the graph is planar. You may deduce this by determining if any edges **must** cross.

[Total: 6 marks]